ME-221
PROBLEM SET 3

Problem 1

Consider the mechanical system shown in Figure 1. A DC motor is connected to a pinion rack mechanism with a mass-damper load. The current flowing through the motor control circuit $i_m(t)$ with resistance R_m is supplied by the input voltage source $u(t)$ and the output of the system is the velocity $v(t)$ of the mass M. The motor-torque and back-emf (denoted by E_m) constants are given by K_t and K_m, respectively. The motor has an inertia denoted by J and the angular velocity of the shaft as well as the gear (with radius r) is given by ω. The parameters f_R and f_T represent the viscous damping coefficients for the motor shaft and the mass-damper unit, respectively.

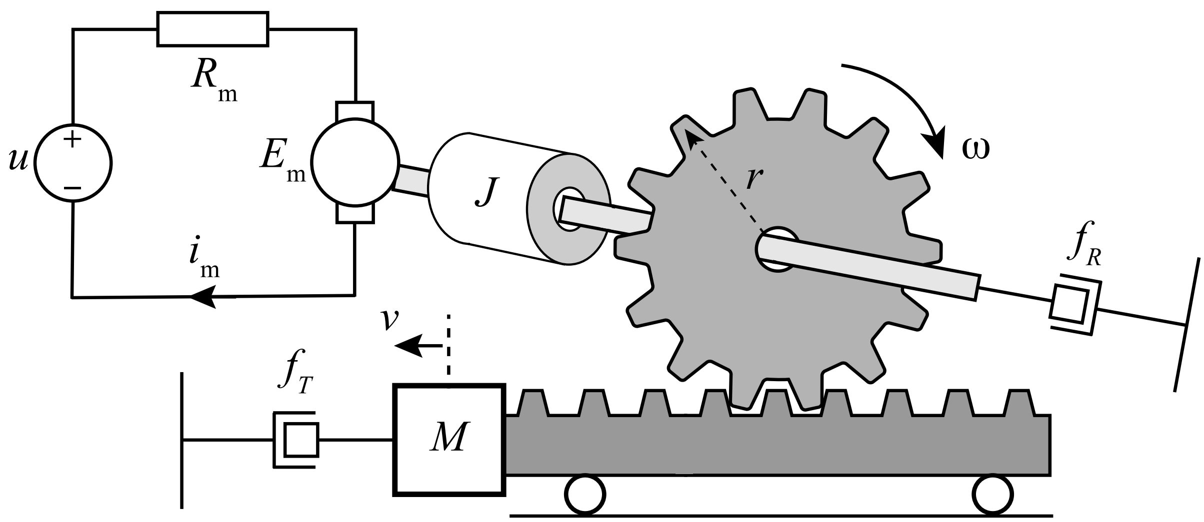

Figure 1: Rack and pinion mechanism driven by an electric motor

a) Derive the equations of motion.
b) Comment on the underlying assumptions and linearity of the model.

Problem 2

A block of mass m is held motionless on the frictionless plane of a wedge of mass M and angle of inclination θ as shown in Figure 2. The plane rests on a frictionless horizontal surface. The block is released.

a) Use the Lagrangian method to derive the horizontal acceleration of the wedge. Hint: Impose a condition of contact between the block and the wedge.

b) Note that we solved this problem using Newton's method in the previous problem set (Problem 2.5). Compare the two solutions.

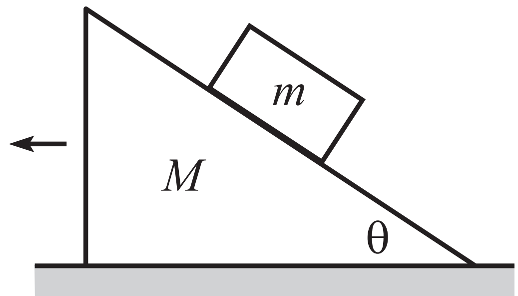

Figure 2: Moving plane

Problem 3

A mass M is free to slide along a frictionless rail. A pendulum of length l and mass m hangs from M as shown in Figure 3. Let x be the coordinate of M and θ be the angle of the pendulum. Find the equations of motion using the Lagrangian method.

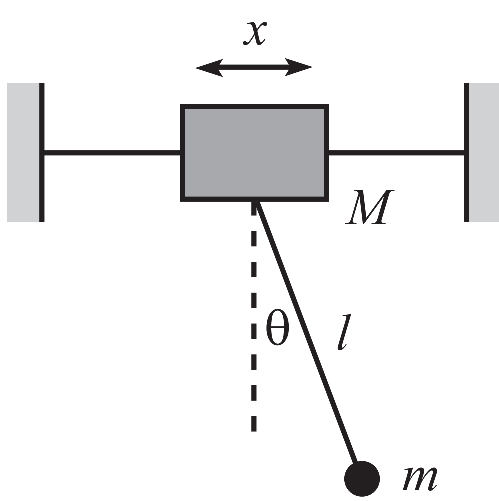

Figure 3: Pendulum with free support

Problem 4

A mass-spring-damper subsystem is located inside a container of mass m_1 as shown in Figure 4. The container of height h is free falling in the vertical direction only. The subsytem consists of mass m_2, linear spring of constant k, and linear viscous damper of coefficient f. Derive the equations of motion for the system using x_1 and x_2 as generalized coordinates, where x_1 and x_2 are the positions of m_1 and m_2 with respect to the ground, respectively. Consider that the spring is at rest when $x_1 = x_2$.

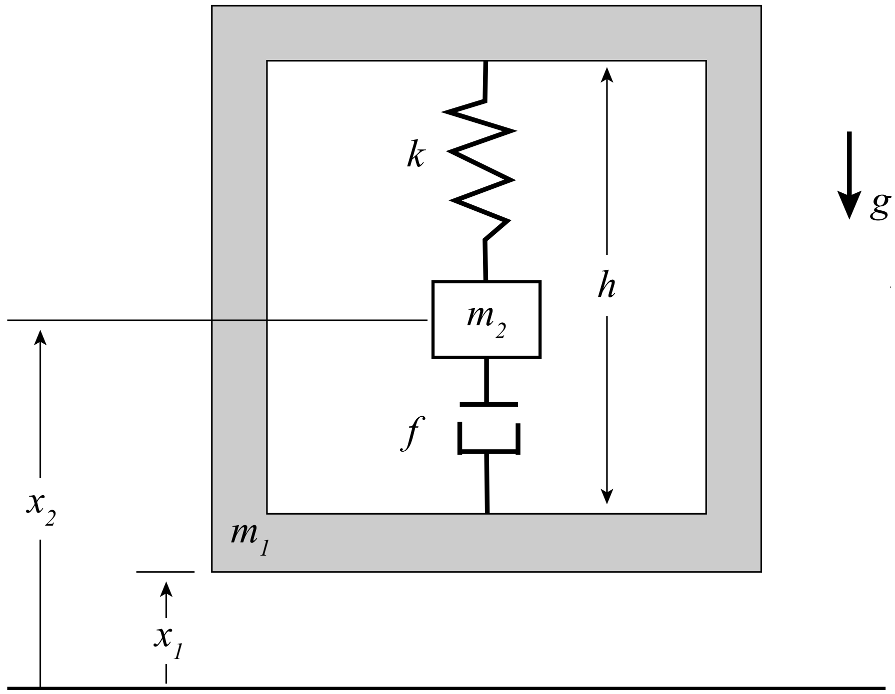

Figure 4: Mass-spring-damper system